

## A study of school mathematics curriculum enacted by teachers in Singapore secondary schools

The “Enactment Project” is a Programmatic Research Project funded by the Ministry of Education, Singapore, and administered through the Office of Educational Research, National Institute of Education, Nanyang Technological University. The project began in 2016 and its aim is to study the enactment of the Singapore mathematics curriculum across the whole spectrum of secondary schools within the jurisdiction. Under this overarching goal, there are two supporting studies: Study 1 examines the classroom enactment by teachers in relation to the curriculum framework as organised in the Pentagon (Skills, Concepts, Attitudes, Processes, Metacognition, with Problem Solving at its centre); Study 2 focuses on the enactment as seen through the instructional materials designed by the teachers.

**Chair/Discussant:** Berinderjeet Kaur

**Paper 1:** Toh Tin Lam, Berinderjeet Kaur, Tay Eng Guan, Lee Ngan Hoe, & Leong Yew Hoong *A study of school mathematics curriculum enacted by teachers in Singapore secondary schools.*

This paper provides an overview of the study, which covers the background, the organisation into two supporting studies, the methodology, and the phases of the project.

**Paper 2:** Berinderjeet Kaur, Lee Ngan Hoe, Ng Kit Ee Dawn, Yeo Boon Wooi Joseph, Yeo Kai Kow Joseph, & Liyana Safii *Instructional Strategies Adopted by Experienced Secondary Teachers when Enacting the Singapore School Mathematics Curriculum.*

This paper presents preliminary findings of Study 1. In particular, it examines the instructional strategies adopted by teachers in the first phase of the project – where thirty competent teachers were selected for close study, which included video-recording of a suite of lessons and post-lesson interviews.

**Paper 3:** Leong Yew Hoong, Cheng Lu Pien, & Toh Wei Yeng Karen *Chronologically-grounded survey.*

This paper describes a methodological contribution by Study 2. From Phase 1 of the project, we obtained some characteristics of design utilised by competent teachers. To study the extent in which these characteristics capture the design work of teachers across Singapore secondary schools, we developed an instrument: Chronologically-grounded survey.

**Paper 4:** Tong Cherng Luen, Tay Eng Guan, Berinderjeet Kaur, Quek Khiok Seng, & Toh Tin Lam *Singapore Secondary Mathematics Pedagogy: The DSR DNA.*

This paper reports findings from a statistical analysis of a survey on 689 teachers in the second phase of the project. In particular, it analyses data from 32 items in one component of the survey regarding teacher moves in the classroom.

# A study of school mathematics curriculum enacted by teachers in Singapore secondary schools

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This paper provides an overview of a programmatic research project, the Enactment Project, at the National Institute of Education in Singapore. The project studies the enactment of school mathematics by competent experienced teachers in secondary schools and also how widespread practices of these competent experienced teachers are in the classrooms of mathematics teachers in general. It elaborates the two studies and two phases of the project.

## The Enactment Project

### *Background*

The main goal of this programmatic research project, the Enactment Project, is to examine how experienced secondary school teachers implement the designated curriculum prescribed by the Ministry of Education in the 2013 revision of curriculum. This research is timely as it will be carried out in 2016-18, three to four years after the revised curricula for mathematics has been introduced. The findings will be pertinent for subsequent revision of the curricula.

It is a special focus project of system studies in pedagogical and educational outcomes. It focuses on understanding what goes on and what works in Singapore's classrooms – more specifically, the instructional core (City, Elmore, Fiarman & Teitel, 2009). The instructional core comprises

“the teacher and the student in the presence of content ... it is the relationship between the teacher, the student, and the content – not the qualities of any one of them by themselves – that determines the nature of instructional practice, [even though] each ... has its own particular role and resources to bring to the instructional process” (City et al., 2009, pp. 22-23).

The project is about the interactions between secondary school mathematics teachers and their students, as it is these interactions that fundamentally determine the *nature* of the actual mathematics learning and teaching that take place in the classroom. It also examines the content through the instructional materials used – their preparation, use in classroom and as homework. Such studies are crucial for the Ministry of Education (MOE) in Singapore and schools to gain a better understanding of what works in the instructional core in their classrooms and schools. This is critical for the development of their education system.

Past large scale studies (Hogan et al., 2013a, 2013b, 2013c) of mathematics teaching and learning in Singapore secondary classrooms, involving random samples, suggest that:

- i) Teachers focus more on procedural knowledge than conceptual knowledge and only engage students in domain-specific knowledge practice in about a third of the instructional time of a typical lesson. Procedural learning support is evident as teachers often help students with “how to do steps”.
- ii) Students are engaged in doing performative tasks (77.3%) more than knowledge building tasks (22.7%). A performative task mainly entails the use of lower order thinking skills such as recall, comprehension and application of knowledge while a knowledge building task calls for higher order thinking skills such as synthesis, evaluation and creation of knowledge.
- iii) The dominant performative orientation of pedagogical practice in Singapore (Hogan et al., 2013c, p. 100) may explain Singapore’s stellar performance in international studies.

While the above findings provide some insights about the widespread orientation of secondary school mathematics classroom teaching and learning in Singapore, they do not inform us about what our competent experienced teachers do when compared to the broad base of teachers studied. Furthermore, it is also not possible to infer how the “performative orientation” has contributed to Singapore students’ performance in PISA studies.

### *Conceptual framework*

Conceptualization of the curriculum enactment process is shaped by the visual model, shown in Figure 1, created by Remillard & Heck (2014). Kaur (2014) noted that this model was rigorous for researching the enactment of school mathematics curriculum in Singapore.

The model shows that as teachers draw on the designated curriculum (which in the case of the project is the Mathematics Syllabus for Secondary Schools (Ministry of Education, 2012)) along with other resources (particularly instructional materials) to design instruction they create what we would refer to as “teacher-intended” curriculum in the context of the project. It includes the interpretation and decisions teachers make to envision and plan instruction. Remillard and Heck (2014) noted that this form of curriculum is difficult to document as part of it exists in the most detailed form in the teacher’s mind. Nevertheless, detailed teacher plans and post lesson video stimulated interviews with the teachers may offer an opportunity to capture the teacher-intended curriculum and its enactment succinctly.

### *The two studies*

The two studies of this project are:

- i) Study 1: Pedagogies adopted by competent experienced mathematics teachers when enacting the curriculum
- ii) Study 2: Competent experienced secondary school mathematics teachers’ use of instructional materials for the enactment of the curriculum

Each study is guided by a string of detailed research questions which are detailed in Kaur et al., (2018). Both studies draw on the same data set and are being carried out concurrently.

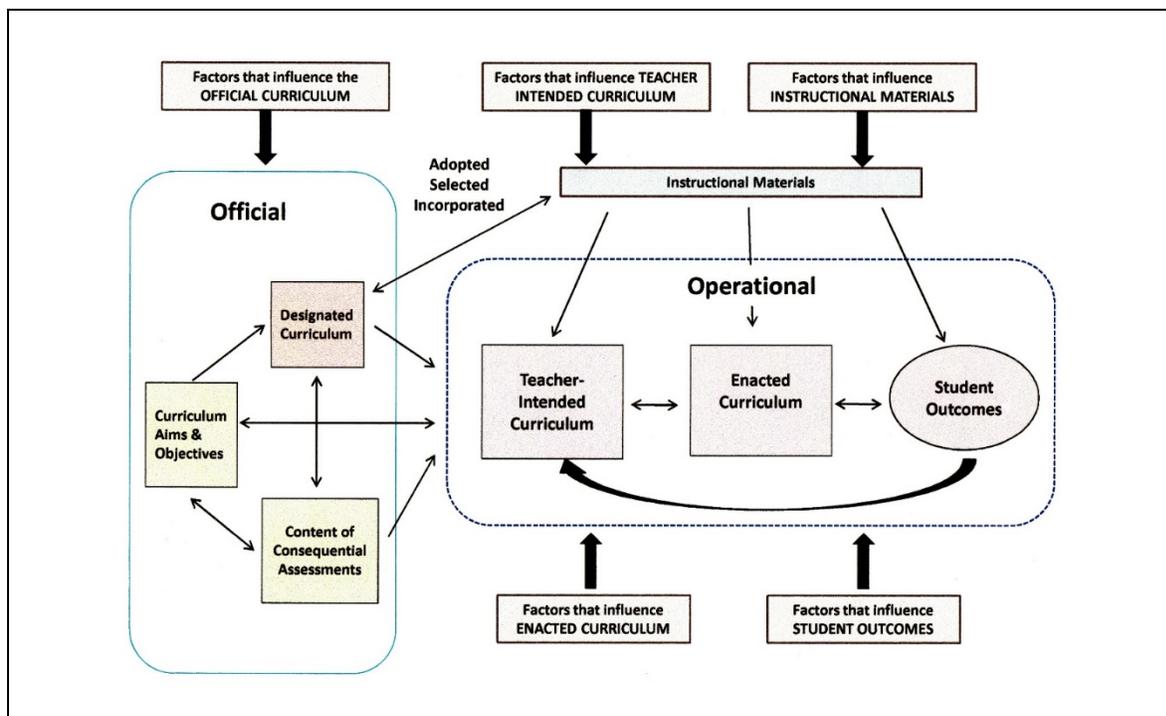


Figure1: Model of the curriculum enactment process (Remillard & Heck, 2014, p. 709)

### *The two phases*

The project has two phases, the first and second. The first phase is the video segment and the second one is the survey segment. The survey segment is dependent on the findings of the video segment. The video segment documents the pedagogy of competent experienced secondary mathematics teachers while the survey segment aids in establishing how uniform the pedagogy of these competent experienced teachers is in the mathematics classrooms of Singapore schools. The video segment of the study is adopting the complementary accounts methodology developed by Clarke (1998 & 2001), a methodology which is widely used in the study of classrooms across many countries in the world as part of the Learner's Perspective Study (Clarke, Keitel & Shimizu, 2006). This methodology recognizes that only by seeing classroom situations from the perspectives of all participants (teachers and students) can we come to an understanding of the motivations and meanings that underlie their participation. It also facilitates practice-oriented analysis of learning. For the survey, the project is adopting a self-report questionnaire to collect data on teachers' enactment of their "teacher-intended" curriculum.

Thirty competent experienced teachers (10 Express course of study, 4 Integrated Programme, 8 Normal (Academic) course of study and 8 Normal (Technical) course of study) and approximately 600 (in each class about 20 students, who volunteered to be the focus students were interviewed) students in their classrooms participated in the video segment of the project. In the context of the project, a competent experienced teacher is one who has taught the same course of study for a minimum of 5 years, is recognized by the school / cluster as a competent experienced teacher who has developed an effective approach of teaching mathematics. These teachers were nominated by their respective school leaders and the research team followed up on the nominations and interviewed the teachers. A strict requirement for participation in the study was that the teacher had to teach the way she / he

did all the time, i.e., no special preparation was expected.

For the survey segment of the project, 690 secondary school mathematics teachers, purposefully sampled and representative of the profile of mathematics teachers in Singapore secondary schools, participated in the project.

### *Data and findings*

The following three papers in this symposium provide some results from phases 1 and 2 of the project.

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# Instructional Strategies Adopted by Experienced Secondary Teachers when Enacting the Singapore School Mathematics Curriculum

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Classroom instructional quality is seen as crucial to student learning. Of particular interest is the interpretation and enactment of the intended mathematics curriculum in Singapore schools by experienced mathematics teachers towards quality learning outcomes. This paper discusses the findings from the study which investigate the instructional strategies that are adopted by experienced mathematics teachers based on the intended Singapore school mathematics curriculum. Findings from the study provide important implications for educational policy makers and practitioners on improving teaching and learning.

Mathematical problem solving has been a focus of mathematics learning in Singapore schools since 1990 and has served as a frame of reference for the enactment of mathematics curriculum (Ministry of Education, 2012, p.14). Instruction in mathematics classrooms in Singapore is guided by a problem solving framework that encompasses a strong foundation on what is to be taught and learned in schools. This includes effective classroom instruction that is supported by three phases of learning (Ministry of Education, 2012, p.22): setting up the stage for students to learn (Readiness), fostering student's active role in learning (Engagement), and consolidating acquired knowledge and stretching student's thinking capacity (Mastery). When planning for classroom instruction, a typical lesson could be mapped out from these learning phases into four distinct phases of lesson (Lee, 2009):

1. Introduction, in which teachers foster student readiness by checking for mastery of pre-requisite knowledge and using motivating contexts (Readiness)
2. Development, in which teachers teach for attainment of the objective of the lesson (Engagement)
3. Consolidation, in which teachers provide opportunities for students to practice on tasks related directly to the objective of the current lesson (Mastery)
4. Closure, in which teachers summarise the lesson and assign homework or follow-up activity to set the stage for the next lesson (Mastery)

The Singapore School Mathematics Curriculum Framework highlights that fostering student problem solving competencies is dependent on five inter-related aspects: conceptual understanding, skills proficiency, mathematical processes, metacognition and attitudes (Ministry of Education, 2012, p.14). These five inter-related aspects support teachers in building a learning environment which is bounded by elements such as student-centredness, and diversity and creativity in learning. This framework has been sustainable, with little modification made to it, since its development in 1990 due to its rigour and robustness in

highlighting the philosophy and principles that influence decisions on what students should be equipped with within our mathematics education (Lee, 2008).

The intent of a curriculum framework is to set parameters for providing high quality learning opportunities to students while allowing some variations on how curriculum is enacted, on the condition that the prescribed standards are adhered to (UNESCO International Bureau of Education, 2017). This suggests that how the phases of lesson unfold in the classroom is extensive and largely dependent on the translation of the intended curriculum prescribed at the national level into the implemented curriculum characterised by instructional strategies adopted by teachers. While past studies have provided us with insights on general characteristics of mathematical instructional strategies in Singapore classrooms (e.g., Hogan et al., 2013; Hyun & Lee, 2017), there is a dearth in the investigation of how instructional strategies adopted in Singapore secondary schools compare to our robust School Mathematics Curriculum Framework.

This study is part of a programmatic research study that examines the instructional strategies and materials that experienced teachers adopt when enacting the intended mathematics curriculum in Singapore secondary schools. This paper reports preliminary findings on the instructional strategies that experienced mathematics teachers adopt as they enact the five aspects of problem solving prescribed in the Singapore School Mathematics Curriculum Framework.

## Methodology

The study reported here is part of a larger study which consists of two phases. This study, which is the first phase of the larger study, involved 30 secondary mathematics teachers from 23 different schools who volunteered to participate in the study after being nominated. The purpose of this phase was to document the instructional strategies employed by experienced teachers. The teachers in our sample were recommended by their schools or professional community based on two requisites: (a) having taught the same course of study for at least 5 years, and (b) being recognised as having developed effective instructional strategies for his/her mathematics lessons.

Secondary education in Singapore is based on four different courses of study and our participants were selected such that our sample was representative of teachers from all the courses of study (see Table 1) as follows:

The second phase of the larger study, which will be reported elsewhere, examined how widespread the instructional strategies adopted by experienced teachers is in mathematics classrooms in Singapore.

Table 1  
*Number of Participants Teaching in Each Course of Study*

Course of Study	Number of Participants
Integrated Programme (IP)	4
Express	10
Normal Academic (NA)	8
Normal Technical (NT)	8
Total	30

This paper focuses on presenting the preliminary findings from Phase 1. Here, teacher instructional strategies were mainly documented through lesson observations, which was also video recorded. The design for the video recording sessions was adopted from Clarke's

(1998) complementary accounts methodology. Each teacher was recorded for a series of consecutive lessons that complete one mathematics topic. The lessons were recorded by three video cameras: (a) teacher camera that focused on how the teacher delivers the lesson, (b) student camera which captured the actions of two selected students seated adjacent to each other, and (c) whole class camera that recorded the overall behaviour of the class. For the purpose of this paper, only recordings captured by the teacher camera would be analysed.

In total, 211 lessons that were video recorded were coded for analysis by a research team comprising experienced and knowledgeable mathematics educators. The data was coded with reference to the five aspects of problem solving prescribed in the Singapore School Mathematics Curriculum Framework (Ministry of Education, 2012, p.14): concepts, skills, processes, metacognition and attitudes.

## Findings & Discussion

*Introducing/ constructing Mathematical concepts:* Our findings revealed that the experienced teachers in our sample adopted a cognitivist approach where the focus is on the assimilation of new information with student's prior mathematical knowledge and the accommodation of new information into student's pre-existing mathematical knowledge (Wadsworth, 1996). This was observed when teachers ( $n = 22$ ) used a whole-class explanation approach to introduce students to new concepts in the classroom, while posing questions along the way. Teachers ( $n = 18$ ) also employed a constructivist teaching approach (Bruner, 1961) by guiding the class or assigning students with tasks which serve to help them actively discover the concepts for themselves. These forms of classroom instruction include elements of cognitive engagement that enable students to understand how mathematical processes work, and the underlying relationships or connections that are crucial to problem-solving proficiency.

*Developing students' fluency in skills related to computing or manipulating mathematical tasks:* It appears that teachers deliver their lessons through single or multiple cycle(s) of development-consolidation phases, which involved cycles of teaching and review of student work. Half of the participants ( $n = 15$ ) conducted their lessons in multiple cycles of development and consolidation of concepts or skills. These multiple cycles were observed within one lesson and within a series of consecutive lessons in which a whole chapter was taught. On the other hand, single cycle of development and consolidation of concepts or skills was used by 14 teachers when delivering lessons. Teachers ( $n = 24$ ) also explained or showed the solution of one or a few worked example(s) prior to giving students questions for practice. The teachers placed emphasis on engaging students with mathematical concepts and provided opportunities for students to achieve mastery in learning through follow up practices which reinforce their understanding of new knowledge. These cycles are reflective of the engagement and mastery learning phases specified in our curriculum framework (Ministry of Education, 2012).

*Emphasizing mathematical processes and developing students' metacognitive strategies:* Teachers ( $n = 18$ ) made thinking visible to encourage articulation of reasoning and metacognitive awareness by presenting students with opportunities to explain and justify their solutions or their peers' solutions to a problem. Teachers ( $n = 13$ ) also fostered metacognitive regulation by making students compare different ways of solving a mathematics problem.

*Imbuing desired attitudes towards Mathematics learning:* Teachers addressed the affective aspects of learning by (a) giving students tasks that they were competent at before progressing to more challenging tasks to foster their confidence in doing mathematics ( $n = 16$ ), and (b) encouraging perseverance in their mathematics learning ( $n = 13$ ).

The abovementioned instructional strategies were found to be demonstrated fairly consistently across all courses of study, except the imbue ment of desired learning attitudes. Relatively lower proportion of IP teachers assigned time and resources towards building positive student learning attitudes as compared to teachers from the other three courses of study. This variation suggests that teachers deem it necessary to serve different student needs more efficiently; teachers saw a need to develop positive self-concepts that could bolster the learning process for students who are not in the IP, whereas students in the IP could have been seen as self-sufficient in this aspect.

## Conclusion

Setting it apart from past studies, this study scrutinised how experienced teachers in Singapore direct their lessons to maintain high quality of mathematics learning that is guided by a robust framework. Our findings revealed that the instructional strategies established by our experienced teachers are aligned to the intended objectives prescribed by our problem solving framework. These insights have implications on policy makers and practitioners on using a curriculum enactment framework to guide pre-service teachers to evaluate and reflect on their teaching practices as beginning teachers, and to improve instructional strategies for in-service teachers.

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## Chronologically-grounded survey

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This is a common experience among education researchers: we notice a surprising instructional move enacted by a teacher. After deeper reflection, we ask, “How many other teachers in Singapore do this?” In this paper, we describe an instrument we designed that is part of the resource needed to answer such questions.

In this paper, we focus on the challenge of constructing survey items for the purpose of eliciting teachers’ commitment to particular instructional practices. This enterprise encounters these theoretical hurdles: How do we strengthen the match between the meaning the crafter imbues into the item and the meaning that teachers draw from the item? How do we strengthen the match between the issues-of-focus that are of interest to the crafter of the items and the issues-of-focus that are of interest to teachers? How do we strengthen the match between the teacher’s commitment to the survey items and his/her commitment in actual practice? Historically, each of these questions corresponds roughly to the respective problems of construct validity, research-practice link, and the gap between avowed belief and actual practice. We will return to these after the next section.

### Background

The study reported here is part of a bigger research enterprise known as the Enactment Project. The aim of the project is to find out how the mathematics curriculum is enacted in Singapore secondary schools. We proceeded in two phases. Phase 1: we conducted in-depth analyses of classroom instructional practices adopted by thirty exemplary teachers. We derived common characteristics across some of these teachers. Our findings in this phase are reported elsewhere (e.g., Leong, Cheng, Toh, Kaur, & Toh, in press; Leong, Kaur, Lee, & Toh, in press) and will not be repeated here. Phase 2: we sought to examine the extent of these characteristics across more secondary schools in Singapore. We targeted 600 teachers from a wide range of schools. In terms of realistic methodology, it is not feasible to proceed using the same research instruments and frames – that is, video-recording of representative lessons with teacher- and student-interviews interspersed within these recorded lessons – at this larger scale. As such, we drew the data in this second phase from teachers’ response to an online questionnaire. The survey items from this questionnaire drew from the characteristics we derived in Phase One. This challenging ‘transference’ from characteristics-of-practice in Phase One to characteristics-to-survey in Phase Two is the subject of our investigation in this paper.

We limit the scope to characteristics with respect to teachers’ design of instructional materials. By instructional materials (IM), we mean materials that teachers bring into the classroom for instructional purposes, and in a form that is classroom-ready for students’ access in the learning of mathematics.

## The survey items

The purpose of the survey items is to determine the extent to which Singapore secondary mathematics teachers utilise certain moves in designing their IMs. How do we translate these moves we extracted from the earlier teachers into survey items in such a way that other teachers who read the items can resonate with its contextual meanings when responding to the questionnaire? Chronologically-grounded survey (CGS) is an attempt at addressing this question.

The CGS carried these features: (i) it was based on the actual IM used by teachers in Phase One; (ii) it presents (in the first section of the questionnaire) a broad sweep of how the IM was used by showing chronological snippets of the IM; (iii) the items (in the second section of the questionnaire) zoom-in to specific moves used by the teacher in each snippet to elicit response. Figure 1 gives an extract of one such online screen-page in the second section of the questionnaire.

**Characteristics in Survey**

- Recap
- Connect
- Skill
- Template
- Formula
- Context
- Think**
- Challenge
- Practice

**Lesson 7: Think Beyond a Method**

**Task:** Examine each of the following questions. Discuss with your partner and decide which method you prefer to solve each of the following questions. Justify your choice.

	Question	Your Chosen method	Reason(s) for Your Choice
1.	Find the roots of the equation $2x^2 - 5 = 9x$ .		
2.	Solve $2x^2 + 9x - 15 = 0$ , giving your answers correct to 2 decimal places where necessary.		
3.	Solve the equation $2(x - 5)^2 = 100$ .		

**Your instructional materials on "Think" reflect this design move.**

	Never/Rarely	Sometimes	Frequently	Mostly/Always
Provide tasks for students to compare methods	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
Press students to justify	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Figure 1. Extract of one page of the online CGS

Figure 1 provides a concrete illustration of the three features: (i) the diagram at the top-right is an extract from a teacher's IM; (ii) the left column retains the chronological categories that were covered in the first section of the questionnaire; and (iii) the response items at the bottom describe the zoomed-in design moves relevant to this particular part of the teacher's IM.

We think that CGS has the potential of addressing the challenges we raised in the introductory section of this paper: (a) [Construct validity]. The use of authentic IM and the chronological arrangement strengthens the teacher's closeness of interpretation to the intended meaning of the items. Singapore teachers are known to organise their instructional plans and routines along temporal lines. Compared to 'contextless' survey items, it is easier for teachers in the CGS environment to experientially connect to the chronological flow and contents of the IM, thus more readily respond to the items with a degree of mental resonance (or dissonance); (b) [research-practice link]. We did not craft the items based on some imaginary or purely theoretical starting point. Rather, as explained in the preceding section, we drew from the actual IMs used by the teachers, and based on careful analyses, derived

the characteristics of design that are “grounded” in practice. This rigorous process strengthens the closeness-to-practice within the research setup; (c) [belief-practice gap]. When teachers commit to, say, “frequently” to a response item, they are indicating an avowed belief towards the stated design move. It is acknowledged that there can be a significant gap between this avowed belief and actual design moves. With the items being “chronologically-grounded”, there is a higher likelihood that teachers would not merely read these items in abstraction; rather, they would project mental imageries of how these sections of the IM square (or not) with their lived professional experiences.

### From the data

Internally, we discuss at length the tension between heightening teacher’s sense of resonance with the prompts and the accentuation of the social desirability bias. The line of thought is: higher contextual relevance provided by the CGS items may increase the former but also the latter – as teachers envisages more clearly the instructional utility described in the items, wouldn’t they also tend to rate highly on these items as they are then seen to depict socially desirable professional traits? If this is so, then the net effect may not be a strengthening of construct validity, as claimed earlier.

As an initial exploration to this tension, and due to space limitations, the investigation we report here is solely based on responses to one snippet – the page as shown in Figure 1. This was selected for two reasons: (1) Since the label for this page is “Think”, and with the Singapore education authority’s continual emphasis on “thinking schools” over the last two decades, this is likely to be the subject for the greatest social desirability bias effect; (2) Based on qualitative responses in the survey, the snippet as shown in Figure 1 has the highest count in terms of explicit references of positive comments. 42 out of 302 teachers indicated that they liked this page. [302, not 600, because teachers were channeled into different survey tracks according to their teaching experiences]. Again, the logic is: since this is the most-liked page, it is most likely the subject of the social desirability bias.

We looked at the responses of the two items listed under this page (see Figure 1): Item A – Your instructional materials on “Think” reflect this design move [as in, you] provide tasks for students to compare methods; Item B - Your instructional materials on “Think” reflect this design move [as in, you] press students to justify. We assigned the scores of 1, 2, 3, 4, to the responses of “Never/Rarely”, “Sometimes”, “Frequently”, “Mostly/Always” respectively. The summary of means is given in Table 1.

Table 1  
*Summary of means*

Item	All 302	42 who likes
A	2.40	2.26
B	2.47	2.29
All	2.91	2.94

The means show that, for all the 302 respondents, the claim for regular-use of the features flagged in this snippet in their own design of IMs is not strong. In other words, there is no evidence in support of the social desirability bias from the responses to items in this snippet. Moreover, we make a stronger case using this logic: if there were social desirability bias, it would have been more pronounced among the 42 who had explicitly stated that they endorsed this snippet; instead, their means were lower than the overall means of the respective items.

At least, there is no evidence here of such a bias that would threaten construct validity.

An interesting finding – we think a direct consequence of the close resonance that CGS affords to teachers (that is, the research-practice link) – is that, out of the 42 who indicated they liked the page, eight made explicit statements about how they have learnt from this particular piece of IM. Language such as “I want to adopt ...”, “I will like to try ...”, “I am intrigued by ...” were used. We extract this particular response because it captured the admirable honesty necessary for professional growth that we advocate: “Regretfully, I don’t encourage or provide students [with opportunities] to explore alternative methods, let alone compare them. I need to work on this ...”. If this honesty can be shown to be pervasive across the sample (an effort beyond the scope of this paper), it will support CGS’s facility in closing the belief-practice gap.

That IMs can be “educative” (Davis & Krajcik, 2005) – in the sense that teachers can learn professionally when using them – is not surprising; that the CGS can be a platform to facilitate this educative encounter is encouraging. This can open up new ways of conceptualising professional development programmes – perhaps, integrate a teacher survey component using CGS into in-service courses?

## Discussion

CGS is in the early stages of instrumentalisation, and the ideas purported here are exploratory. But we think CGS can potentially contribute to the development of surveys that are practice-near without compromising the aims and rigours of research. We are not aware of teacher survey instruments of this nature that are currently administered at scale. Research on “what teachers really do in the classroom” can be broadly classed as: (1) case studies based on analysis of actual classroom observations or videos. Due to logistical resources demanded, realistically, only a few cases can be studied this way. This challenges any claims of representativeness of the findings to the wider jurisdiction; (2) survey designs that targets large sample size for representativeness. This overcomes the logistical constraints but usually compromises the “resonance” with ground experiences. The results of the research are generally ‘distant’ and irreconcilable with “what teachers actually do in the classroom. CGS is a bridge between these two traditional paradigms of research: we began with case studies to derive the characteristics, and use them to craft survey items that are “chronologically-grounded”.

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# Singapore Secondary Mathematics Pedagogy: The DSR DNA

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*The Enactment Project* seeks to find out in more detail what is happening in Singapore mathematics classrooms. In particular, the video data of 30 teachers suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA, which is characterized by cycles of Development, Seatwork and Review (DSR). This paper reports findings from the statistical analysis of a survey on a further 689 teachers regarding aspects of the DSR.

In this paper, we attempt to further explore the instructional core, comprising aspects related to three main components, Development, Seatwork and Review (Kaur, 2017). Our in-depth study of sequences of lessons of 30 competent secondary school mathematics teachers suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA. Teachers develop conceptual knowledge through a myriad of ways, before engaging students in seatwork to consolidate their learning which is followed by review of student work in class which allows errors to be springboards for deeper understanding of knowledge explored during the lesson. Each instructional cycle is guided by a micro-instructional objective. Through such cycles the objective of the lesson is carefully achieved, ensuring that conceptual understanding and procedural fluency are achieved through both teacher-centred and student centred activities.

## Background

The study reported here is part of a bigger research enterprise known as the Enactment Project. Leong, Cheng and Toh (2019) in another paper in this symposium series explained the two-phase research method as follows: In-depth analyses of classroom instructional practices adopted by thirty exemplary teachers, from which common characteristics were derived, and a survey of 689 teachers to ascertain the extent of these common characteristics. The video data from the first phase suggests that there is a distinctive Singapore secondary mathematics pedagogy, almost like its DNA, which is characterized by cycles of Development, Seatwork and Review (DSR). This paper reports findings from the second phase regarding aspects of the DSR.

## Participants and Instrument

The participants were Singapore secondary school teachers from 4 different academic courses: Integrated Programme (60), Express (388), Normal Academic (151), and Normal Technical (90). These four academic courses are broadly based on academic achievement in the Primary School Leaving Examination.

An on-line questionnaire was administered to these teachers with their written consent. The questionnaire consisted of 3 sections: pedagogical structure and student-teacher interaction (60 items); enactment of the different facets of the “pentagon framework” (MOE, 2012) undergirding the secondary mathematics curriculum (78 items); and instructional materials (226 items, of which a participant needs only respond according to one of the subjects Additional Mathematics, Elementary Mathematics, Normal Academic Mathematics).

The 60 items in the first section were divided into two parts of 36 and 24 items respectively.

The first part had items which elicited responses about what the teacher did in class. Three sample items follow below:

1. I focus on mathematical processes (such as compare and contrast, logical reasoning) to facilitate the development of concepts or student understanding
2. I engage students in practising past exam papers
3. I provide collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework

The second part had items which elicited responses about what the teacher wanted the student to do in class. Three sample items follow below:

1. I get my students to explain how their solutions or how their answers are obtained
2. I get my students to practise a similar problem after you have shown them how to do it on the board
3. I get my students to critique one another's work presented on board/screen so as to improve their understanding of concepts or elegance in their presentation/solution

The 60 items were also constructed around ‘teaching moves’ of Development, Student Seatwork and Review. The sample items above are, in order, of each type. Participants were required to respond on a Likert Scale of 1 (Never/Rarely) to 4 (Mostly/Always). In addition, the items in each type were designed to reflect two main teaching styles, namely direct teaching for fluency (Fluency) and student-centred teaching (Student-Centred).

In this paper, we shall only discuss findings based on the first part of the pedagogical structure and student-teacher interaction section. We removed 3 items which focused on whether the teachers used the textbook or customized worksheets as these could be interpreted both as Fluency and Student-Centred. We shall refer to the remaining 33 items as ‘the instrument’.

## Data analysis

Generally, satisfactory reliability and validity of the instrument were established. Means, standard deviations, and Cronbach’s  $\alpha$  of the three subscales are shown in Table 1. According to Hatcher and Stepanski (1994), a threshold of .55 level of Cronbach’s  $\alpha$  can be used in exploratory research. As shown in Table 1, all six subscales demonstrated satisfactory internal consistency.

Table 1  
Means, standard deviations and Cronbach's  $\alpha$

Scale	Item Mean	Item Mean Variance	Cronbach's $\alpha$
Development Fluency (6 items)	3.248	0.049	.709
Development Student-Centred (9 items)	3.002	0.053	.883
Seatwork Fluency (5 items)	2.844	0.117	.722
Seatwork Student-Centred (2 items)	3.119	0.013	.559
Review Fluency (7 items)	3.023	0.036	.765
Review Student-Centred (4 items)	2.336	0.049	.753

A Principal Component Analysis with Varimax rotation method was performed to identify the factor structure of the instrument. Six factors were returned and they accounted for about 54% of total variance. One item was removed because it had loading of less than 0.5 and only on the 6<sup>th</sup> component, which had the least commonality. A Principal Component Analysis with Varimax rotation method was performed again. Five factors were returned this time and they accounted for about 52% of total variance. No further items were removed but the factor structure was further reduced to four factors because the fifth component was significantly less in communality than those preceding it (1.311 compared with 1.723, 2.072, 3.167, and 8.411).

Since it was reasonable to believe that there would be correlations between the factors, a Principal Component Analysis with Promax rotation was performed on four factors with the factor structure shown in the Appendix. The four factors were named *Student-centred in-class learning*, *Teaching and practice for fluency*, *Teacher-led conceptual learning*, and *Teacher-guided student self-directed learning*. Table 2 shows the correlations among the four factors.

Table 2  
Component Correlation Matrix

	1	2	3
2	.252		
3	.507	.361	
4	.391	.127	.216

## Discussion

Referring to Table 1, we see that 4 of the 6 subscales had item averages of at least 3, indicating that the teachers in general performed such moves in their classrooms. These moves were linked to development of mathematical concepts through teacher talk and student engagement, student seatwork and review for fluency. Moves which targeted practice (Seatwork Fluency (Item mean 2.844)) were surprisingly less common than teacher-student interaction during seatwork (Seatwork Student-Centred (Item mean 3.119)). However, review seemed to be more teacher-directed (Review Fluency (Item mean 3.023)) than student centred (Review Student-Centred (Item mean 2.336)).

We shall now consider the four factors that seems to underlie teacher moves in the Singapore classroom. Instead of bifurcating into student-centred versus teacher-directed learning, or fluency versus conceptual learning, we find that these aspects are mixed and matched into four amalgams. The first is student-centred in-class learning. Teachers are student-centred both in the development phase (they ask questions to encourage reasoning,

and build on students' responses) as well as in the seatwork phase (they provide students with probing guidance (open ended questions), and walk around the class noting students' work that would be used to provide class feedback later). The second is teaching and practice for fluency. These all fall under items in Fluency subscales. Examples of these are using "I do, we do, you do" strategy during the development phase, and engaging students in practising past year exams. Next is teacher-led conceptual learning. Again these all fall under items in Fluency subscales but interestingly they are extracted under a different factor. Looking more closely at the items, we can understand why. Whereas the second factor emphasises fluency through thoughtful practice, this third factor emphasises fluency through conceptual understanding. Some items of this factor are focusing on mathematical vocabulary during the development phase, and helping students identify strategies during the review phase. The final factor is teacher-guided student self-directed learning. Indeed, students need guidance to revise on their own outside the classroom. Thus, moves such as getting students to set their own learning goals and working with their peers to make a plan for revision and correction of mistakes, are attempts by the teacher to ensure that learning takes place outside the classroom.

We chose Promax rotation because it allowed us to see the correlations between the factors. We had always felt that fluency learning and conceptual learning are not mutually exclusive, nor student-centred learning and teacher-directed learning. Indeed, from Table 2, the four factors are seen to correlate significantly. "Student-centred in-class learning" and "teacher-led conceptual learning" have a correlation of 0.507. In addition, "student-centred in-class learning" has reasonably high correlations with the other two factors as well.

The discussion above gives us a clearer picture of how the Development-Seatwork-Review cycle plays out in Singapore classrooms. Data shows that these moves within these phases are generally enacted in the classroom. Interestingly, the data also shows that underlying these moves are student-centred considerations towards fluency and conceptual understanding.

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## Appendix

	Components			
	1	2	3	4
<i>There are several approaches that we may adopt in our mathematics lessons. Reflecting on my lessons, I ...</i>				
use "I do, We do, You do" strategy: Demonstrate how to apply a concept/carry out a skill on the board [I do] Demonstrate again using another similar example but with inputs from students [We do] Ask students to do a similar question by themselves [You do]		0.441		
emphasise basic facts/steps for students to memorise them		0.638		
provide students with sufficient questions from textbooks/workbooks/other sources to practise so as to develop procedural fluency		0.489		
use exposition (teacher at the front talking to whole class) to explain mathematical ideas, facts, generalisations		0.455	0.369	
focus on mathematical vocabulary (such as equations, expressions) to help students build mathematical concepts			0.795	
focus on mathematical vocabulary (such as factorise, solve) to help students adopt the correct skills needed to work on mathematical tasks			0.859	
ask students to recall past knowledge	0.486			
ask direct questions to stimulate students' recall of past knowledge/check for understanding of concepts being developed in the lesson	0.463	0.401		
ask questions to encourage reasoning and speculation, not just to elicit right answers	0.756			
use examples and non-examples to engage students in discussion to make sense of a concept	0.731			
focus on mathematical processes (such as compare and contrast, logical reasoning) to facilitate the development of concepts or student understanding	0.724			
lead whole class discussion (with guided questions) to facilitate the development of concepts	0.737			
exchange ideas with students on how to solve a problem	0.731			
ask students open-ended questions and allow them to build on one another's responses to develop concepts or clarify their understanding	0.846			
build on students' responses rather than merely receiving them	0.763			
get students to automatise steps leading to a solution through repetitive exercises		0.778		
engage students in practising past exam papers		0.681		
provide students with directed guidance (ask close-ended questions) when they face difficulty with a mathematical task they are doing, focusing them on the concept/skill necessary to do the task		0.703		
tell students how to do it when they face difficulty with a mathematical task they are doing		0.717	0.323	
walk around the class and provide students with between desk instruction (i.e. help them with their difficulties) when they are doing work at their desks		0.384		
provide students with probing guidance (open-ended questions about their thinking and why they are considering certain approaches) when they face difficulty with a mathematical task they are doing	0.462			
walk around the class noting students' work that I would draw on to provide the class feedback during whole class review when they are doing work at their desks	0.328			
explain what exemplary solutions of mathematics problems must contain (logical steps and clear statements and/or how marks are given for such work during examinations)			0.581	
help students identify strategies that would help them achieve their learning goals for mathematics			0.461	0.493
encourage students to show me their work and review their progress for mathematics			0.430	0.385
provide feedback to individuals for in-class work and homework to serve as information and diagnosis so that students can correct their errors or improve			0.588	
provide collective feedback to whole class for common mistakes and misconceptions related to in-class work and homework			0.617	
review student performance by providing the class detailed comments on tests and examinations			0.319	0.317
get students to set their own learning goals for mathematics at the beginning of each school term/semester				0.664
get students to make a plan to revise their work and correct their mistakes				0.724
get students to work with peers to make a plan for revision and correction of mistakes				0.734
get students to grade their own mathematics work (with the marking scheme/rubric provided and teach them how to use it)				0.621

Note: Grey rows indicate Fluency subscales.

